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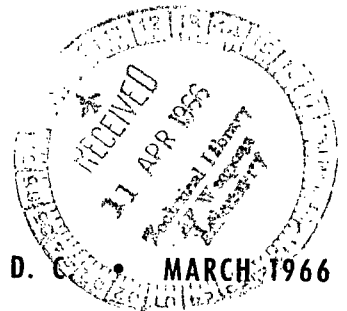
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SPACECRAFT MOMENTUM-EXCHANGE
SYSTEM BY USE OF THE
GRAVITY-GRADIENT MOMENTS

by David B. Middleton
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SUMMARY

An analytical study has been made of attitude control of manned orbiting spacecraft by means of planned momentum exchange between the spacecraft and a system of spinning wheels aboard the spacecraft. This technique allows the spacecraft to maintain a fixed attitude with respect to some quasi-inertial reference (such as the sun) during part of each orbit and then to dissipate the bias portion of the accumulated angular momentum during the remainder of the orbit. The momentum dissipation is accomplished by reorienting the spacecraft in the earth's gravity field in such a way that the unidirectional component of the disturbance torque will be reversed whereas the sinusoidal components will retain their cyclic nature. No attitude jet fuel is required for either the reorientation maneuver or for the momentum dissipation (that is, desaturation of the momentum-exchange devices).

For the conditions studied, it was found that a useful and adequate attitude orientation (for removal of the momentum) can be achieved with a roll-only maneuver during most of the year. For other times, either a modified roll-only maneuver or a variety of multiaxis maneuvers can be used. The multiaxis maneuvers provide a relatively large momentum-dissipation capability and much greater operating flexibility. However, the penalty for this increased capability and flexibility is a significant increase in the size of the momentum-exchange devices and also in the amount of electrical power required to operate them.

The attitude-management tasks associated with the control operations seem to be well suited to manual control. The attitude maneuvers need not be performed in a highly precise manner because small errors in the timing and execution of particular maneuvers can be compensated for during succeeding orbits by the simple process of monitoring and iterative correction.

INTRODUCTION

For manned orbiting spacecraft on extended missions, storage of considerable extra fuel for attitude stability and control is presently considered a spacecraft design requirement. If only the attitude jets were to be used to counteract external torques on a spacecraft, the fuel-storage capacity would have to be very large unless frequent resupply ferries were to be used. Even when momentum-exchange devices, such as inertia wheels and control moment gyros (CMG), are used, a certain amount of attitude fuel is still considered to be necessary for angular-momentum dissipation. (For example, see ref. 1.) The attitude stability and control fuel requirements for several configurations of a Manned Orbital Research Laboratory (MORL) using such devices have been studied under NASA contracts NAS1-2974, NAS1-2975, and NAS1-3612. The weight of this attitude fuel was reported to be between 4448 and 8896 newtons (between 1000 and 2000 pounds) per year for configurations using solar-cell panels fixed rigidly to the space station. The fuel weight was estimated to be somewhat less when gimballed panels were considered. In either case the fuel requirements vary with both altitude and attitude orientation of the space station and, in particular, aerodynamic torques have to be considered in the selection of the most favorable orientations at lower altitudes.

Elimination of the requirement of using attitude-jet fuel for compensation of disturbing torques (such as the gravity gradient) is then a desirable goal if overall mission objectives are not adversely affected. It seems possible to devise appropriate attitude-management plans for many orbital missions if spacecraft orientation requirements can be periodically relaxed. For example, in a space station type of mission, the solar panels must be held approximately normal to the sun's rays during the daylight portion of each orbit to obtain effective conversion of solar energy to onboard electrical power. If the panels are rigidly fixed to the station, the station itself must be held at a fixed inertial attitude during daylight, and the momentum due to the gravity-gradient torque (or other disturbances) can be absorbed by a system of momentum-exchange devices. During the darkness portion of the orbit, the sun-pointing requirement can be relaxed and the station can be reoriented in the gravity field in such a way that the gravity-gradient moments are used to remove any bias components of the accumulated momentum. This approach has been generally overlooked in studies to date.

The present study was undertaken to show that sufficient attitude stability and control can be achieved with a system of momentum-exchange devices and an attitude-management program. Control systems using inertia wheels are simple, highly reliable, and have been adjudged capable of producing adequate spacecraft response characteristics. (See refs. 2 and 3.) For example, the accuracy of attitude pointing an Orbiting Astronomical Observatory (OAO) configuration was determined (in ref. 3) to be within ± 0.1 second of arc when inertia wheels were used for control; for most space-station

experiments, the stability requirements are not so strict. Control-moment gyros (CMG) are also very attractive as control devices primarily because of their smaller power (electrical) requirements. (A system of CMG controls is being recommended for the MORL.) By using either type of momentum-exchange device for attitude control of a space station, the necessity for using fuel-optimum orientations can be eliminated since fuel is not used. It should be noted, however, that the technique considered herein is not aimed at eliminating the attitude jets; they will still be needed for such things as orbit acquisition, orbit keeping, gross attitude changes (for example, emergencies) and possibly spinning up or despinning the station to provide "artificial g" conditions. The primary effect of using the technique will be to reduce the total fuel storage requirements and/or the frequency of resupply ferries.

The momentum-exchange technique considered in this study is not dependent on any particular spacecraft configuration, type of momentum-exchange devices, altitude orbital inclination, pointing direction, or initial orientation of the spacecraft. Data are presented for typical cases involving cylindrical-shaped configurations in orbits inclined 30° to the equator. For convenience, the sun direction was arbitrarily selected as an inertial reference or pointing direction and the gravity-gradient torque is used as a representative continuous disturbance which could cause control problems. By proper choice of reorientation, the technique can also be used to remove bias momentum arising from other types of disturbances.

SYMBOLS

The International System of Units is used in presenting the results of this study. In some cases the English system equivalents are given in parentheses. In case conversion to the English system of units is desired, reference 4 may be used.

a_{ij}	elements in Euler transformation matrix (θ, ψ, ϕ order of rotation)
\bar{g}	acceleration due to gravity, meters/second ²
\bar{H}	angular momentum vector, newton-meter-second
I_{xx}, I_{yy}, I_{zz}	moment of inertia of spacecraft about x_B , y_B , and z_B body axes, respectively, kilogram-meter ²
I_{wx}, I_{wy}, I_{wz}	moment of inertia of X_B -, Y_B -, and Z_B -axis momentum exchange wheels, respectively, kilogram-meter ²

$\hat{i}_B, \hat{j}_B, \hat{k}_B$	unit vectors along X_B -, Y_B -, and Z_B -axis, respectively
l_x, l_y, l_z	direction cosines of the local vertical with respect to X_B -, Y_B -, Z_B -axis, respectively
l_x', l_y', l_z'	direction cosines of local vertical with respect to X_I -, Y_I -, and Z_I -axis, respectively
R	geocentric radius, meters
\hat{R}	unit vector along geocentric radius
t	time, seconds
Δt	reorientation period, seconds
\bar{T}	total gravity-gradient torque, newton-meters (see eq. (1))
X_B, Y_B, Z_B	principal body axes of spacecraft
X_I, Y_I, Z_I	quasi-inertial axes: Z_I -axis always points at center of sun or some other inertial reference point
$\hat{X}_I, \hat{Y}_I, \hat{Z}_I$	unit vectors along quasi-inertial axes
α	one-half the central angle which subtends the darkness portion of the spacecraft orbit, degrees
β	gimbal displacement angle with respect to null position of CMG, degrees
η	orbital travel angle, measured from orbit noon, degrees
ξ	angle that orbit plane makes with sun's rays, degrees
ψ, θ, φ	Euler angles denoting yaw, pitch, and roll, respectively, degrees or radians
ω	angular velocity, radians/second

Subscripts:

e	value of variable at surface of earth
n	conditions at orbit noon
T	total
s	spacecraft
wx	control wheels associated with X_B -axis
wy	control wheels associated with Y_B -axis
wz	control wheels associated with Z_B -axis
max	maximum value of variable
min	minimum value of variable

Dots over variable denote differentiation with respect to time.

Dots between vectors denote dot products.

$||$ denotes absolute value of the variable.

GENERAL CONSIDERATIONS

Some type of efficient stability and control system must be provided for spacecraft on long-term orbiting missions. This system must be capable of orienting and stabilizing the spacecraft (or space station) for various periods of time in the presence of both internal and external torques. The attitude-hold requirements may vary from several seconds to several orbits depending on the function or experiment being performed (for example, pointing, tracking, mapping). During general missions, such as that of a space station, most of the experiments will probably not require particular attitude orientations or precise attitude-hold accuracies. However, some degree of control is necessary at all times just for crew comfort and safety.

External Disturbance Torques

For orbit altitudes around 200 nautical miles, a variety of external disturbance torques are present, but only the gravity-gradient and aerodynamic torques are expected

to cause any stability and control problems. These torques vary greatly with spacecraft configuration and orientation in the orbit. Aerodynamic torques are most significant on configurations which present large frontal areas to the flight direction (for example, a configuration with arrays of large solar-cell panels facing the direction of motion). However, for the higher altitudes and for future configurations which may use power systems based on nuclear technology, only the gravity gradient should cause significant torques on the spacecraft.

Briefly, when a spacecraft is alined in the gravity field so that its principal axes are not along the local vertical, torques will exist about each axis because of the unsymmetrically arranged mass elements. An arbitrary orientation is shown in figure 1 in which R_s represents the local vertical to the spacecraft and R_n represents the local vertical to orbit noon. Also shown is the pertinent orbital geometry relative to a set of quasi-inertial reference axes, X_I , Y_I , and Z_I .

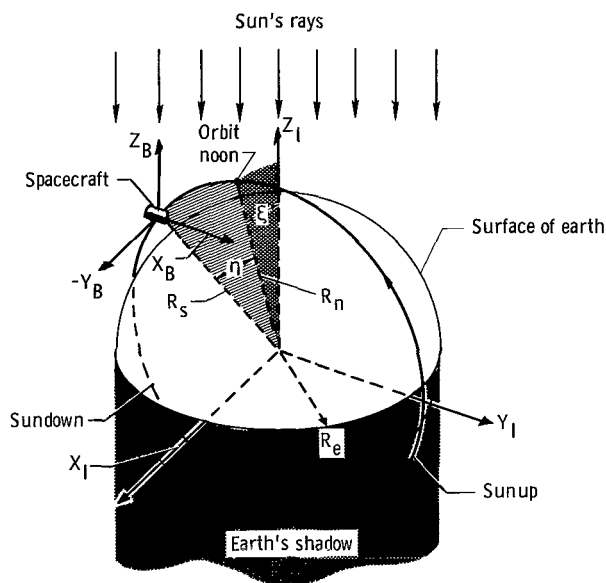


Figure 1.- Orbital geometry and axis systems.

When the roll axis X_B of the spacecraft is alined along the local vertical at "orbit sunup" (fig. 1), the gravity-gradient torque can be expressed in the following simplified form:

$$\bar{T} = K_x \cos^2 \eta \hat{i}_B + K_y \sin 2\eta \hat{j}_B + K_z \sin 2\eta \hat{k}_B \quad (1)$$

where η is the orbital travel angle and K_x , K_y , and K_z are coefficients based on the orbital parameters and the attitude of the spacecraft with respect to the direction of the sun. Orbit sunup and orbit sundown are defined as the points in the orbit where the geocentric radii to the spacecraft are normal to the sun's rays. In equation (1) $\eta = \pm 90^\circ$ at these two points, and thus $\bar{T} = 0$. Actual sunup occurs a few minutes earlier because of the altitude of the orbit and atmospheric refraction of the light rays. For distinction, sunup will be used in this report to refer to actual sunup unless orbit sunup is specified. The sun direction was assumed as a convenient basis of reference for equation (1) and merely represents a typical inertial direction.

A more detailed discussion of the gravity-gradient torque components and some spacecraft reorientation procedures are contained in appendix A.

Momentum Considerations

During periods when the spacecraft is being held at some desired inertial attitude, a set of conveniently located momentum-exchange devices can be used to "absorb" or "store" the angular momentum caused by gravity-gradient torques or other small disturbances. This momentum storage is accomplished by using onboard electrical power to drive the devices in such a manner that an equal and opposite angular-momentum vector is generated to null the effect of the external disturbance. In the case of inertia wheels, the rotational speeds of the wheels are continuously adjusted to provide the momentum-nulling effect. In the case of CMG controls, the spin axes of the gyros are torqued appropriately to provide a resultant momentum vector for nulling. In both cases the following equation applies:

$$\bar{H}_T = \bar{H}_S + (\bar{H}_{wx} + \bar{H}_{wy} + \bar{H}_{wz}) \quad (2)$$

where \bar{H}_S is the angular momentum of the spacecraft and \bar{H}_{wx} , \bar{H}_{wy} , and \bar{H}_{wz} are momenta generated by the control wheels associated with the X_B -, Y_B -, and Z_B -axes, respectively. When an external torque tends to change \bar{H}_S , the total momentum of the system \bar{H}_T can be kept fixed at zero by requiring the appropriate control wheels to provide the following condition:

$$\bar{H}_{wx} + \bar{H}_{wy} + \bar{H}_{wz} = -\bar{H}_S \quad (2a)$$

In a similar manner, the spacecraft can be made to rotate slowly by creating an imbalance between \bar{H}_S and $\bar{H}_{wx} + \bar{H}_{wy} + \bar{H}_{wz}$. High maneuvering rates due to such inputs are probably not feasible for most spacecraft configurations because of the large inertias and power requirements involved. But when time is available for "slow-motion" reorientation, the momentum-exchange devices can provide an economical means of

routine attitude maneuvering. Some considerations for the sizing of momentum-exchange wheels are presented in appendix B.

Basic Assumptions

The study reported herein is general in nature, but the following assumptions were made for convenience and to establish a basis of study:

- (1) The spacecraft is in circular orbit
- (2) The orbit is inclined 30° to the equator
- (3) The earth's gravity field is radially symmetric and a regression motion of the orbital plane is the only effect of the earth's oblateness
- (4) An operational requirement of the spacecraft will be to maintain a constant attitude with respect to a line between the spacecraft and an inertial reference point during most of the time that the reference point is visible to the spacecraft; this requirement may be relaxed when the reference is not visible. The sun was selected as a convenient reference point for this study.
- (5) A system of momentum-exchange devices (inertia wheels and/or control-moment gyros) is available for routine attitude stability and control.
- (6) No fuel will be used for desaturation of the momentum-exchange devices.

RESULTS AND DISCUSSION

As already indicated, equation (1) expresses the gravity-gradient torque for the assumed initial orientation of the spacecraft. Substituting appropriate expressions (see appendix A) for the coefficients, this equation becomes:

$$\begin{aligned} \bar{T} = \frac{3g_e R_e^2}{2R_s^3} & \left\{ \hat{i}_B (I_{zz} - I_{yy}) [\sin 2(\xi - \varphi)] \cos^2 \eta \right. \\ & + \hat{j}_B (I_{zz} - I_{xx}) [\cos(\xi - \varphi)] \sin 2\eta \\ & \left. + \hat{k}_B (I_{yy} - I_{xx}) [\sin(\xi - \varphi)] \sin 2\eta \right\} \end{aligned} \quad (3)$$

where ξ is the angle of inclination of the orbit plane to the sun's rays and the orbital travel angle η is measured from the closest position of the orbit to the sun (or "orbit noon"). (See fig. 1.)

The angular momentum accumulating in the momentum-exchange system is proportional to the areas under the component curves associated with equation (3); these curves for a typical cylindrical-shaped spacecraft are shown by the solid-line curves in figure 2. Strictly, angular momentum is the time integral of the torque, but to aid in visualizing locations of the spacecraft in orbit, η is shown on the abscissa of figure 2 where $\eta = \omega t$ and ω is constant (circular orbits). The torque components are normalized to the magnitude of the maximum total gravity-gradient torque which occurs when the spacecraft is about halfway between orbit sunup and orbit noon and each quarter-orbit thereafter. The Y_B - and Z_B -torques are cyclic but the X_B -torque is biased in one direction. This biased or unidirectional component is generally much smaller (note the expanded vertical scale of the X_B -ordinate in fig. 2) than the other two components but, because of its nature, it creates the primary attitude-control problem for stability and control

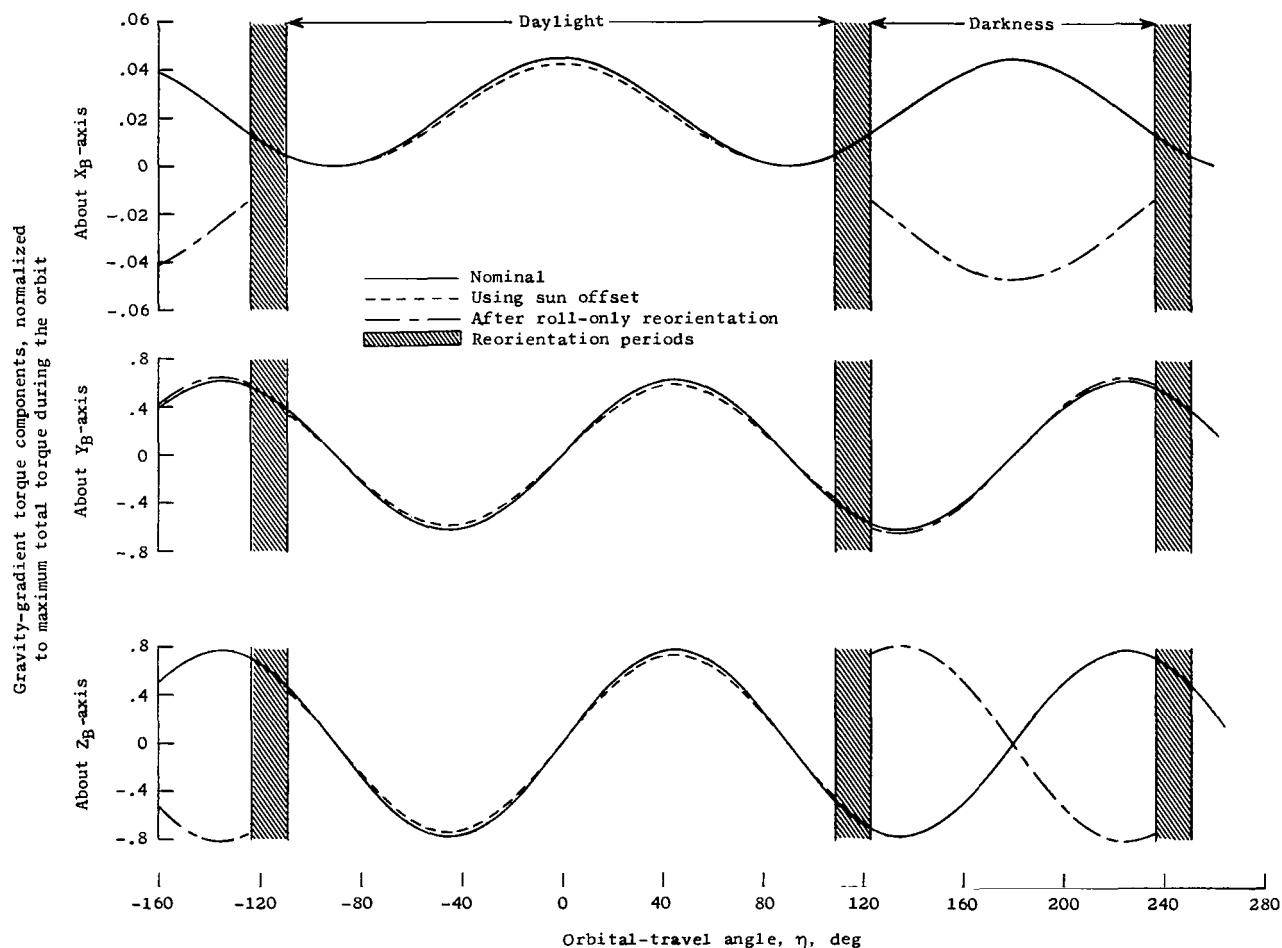


Figure 2.- Normalized gravity-gradient torque components during a typical orbit ($\xi = 53.5^\circ$) showing effects of a sun-pointing offset (3.5°) during daylight and of a spacecraft roll displacement (98.5°) during darkness.

systems using momentum-exchange devices. An external torque to counteract this unidirectional component can be used periodically to prevent a buildup of angular momentum in the system. A nonthrusting scheme to provide this countertorque is considered in the following sections.

Roll-Only Reorientation

For the daylight orientation of the spacecraft, the unidirectional component of the accumulated angular-momentum vector will be in the Y_I -direction (which coincides with the X_B -direction during the accumulation period). Before a saturation condition is reached by the control wheel(s) associated with the X_B -axis, the spacecraft can perform a simple roll-only maneuver by using the wheels themselves as the only control inputs. The beneficial effect of this reorientation in the gravity field is to reverse the direction of the unidirectional gravity-gradient torque component. (See fig. 2.) Note that during the roll-only maneuver, the X_B -axis will remain parallel to the Y_I -direction. The reorientation maneuver also changes the torque pattern about the other two axes. This change is indicated by the long-dash—short-dash curves which lie in the darkness region of figure 2. (The other dashed curves in the daylight region are discussed later in connection with fig. 3.)

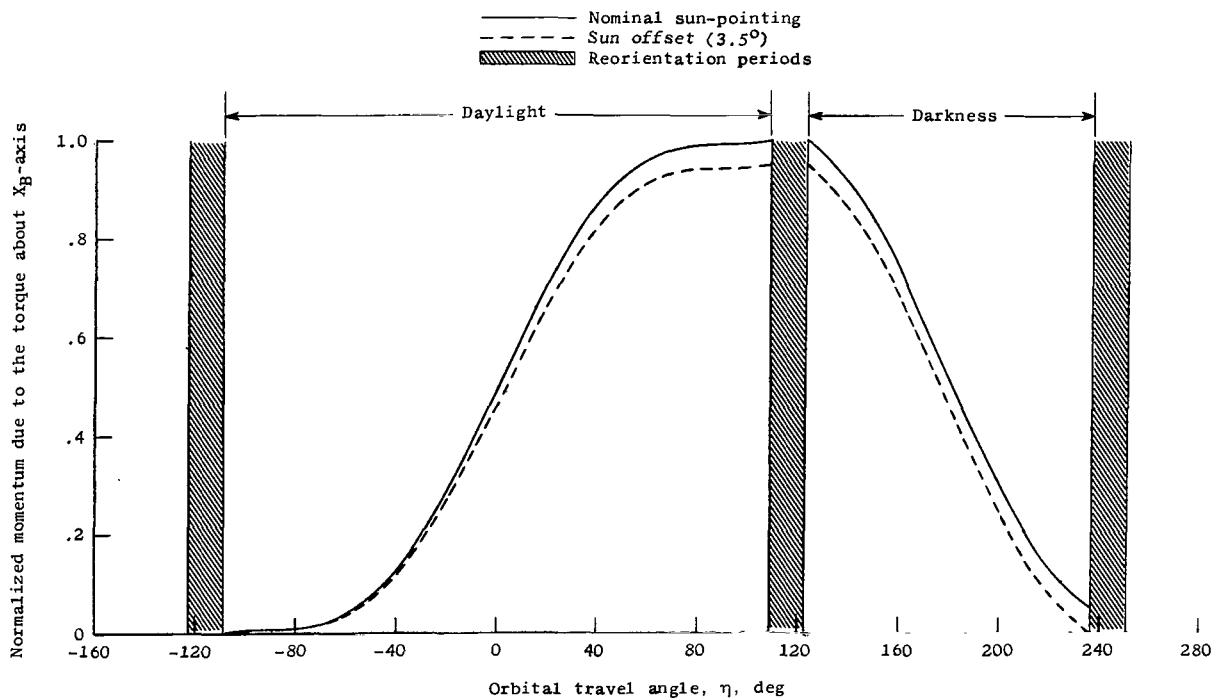


Figure 3.- Momentum accumulation in X_B wheel or wheels during daylight and dissipation during darkness after a roll-only reorientation maneuver.

For the roll-only maneuver, dissipation of the unidirectional component of the momentum is most effective when $\varphi = (45 + |\xi|)$ degrees ($\varphi = 98.5^\circ$ for the case in fig. 2). A comparison of the positive and negative areas under the X_B -torque curve for this case shows that for each unit of momentum which accumulates during the daylight portion of the orbit, about 95 percent can be dissipated during the darkness portion (after a selected reorientation period of 218 seconds). This effect is shown in figure 3 where the net momentum accumulation during reorientation is considered to be zero since the torque curve in figure 2 must cross through zero about the middle of the reorientation interval. Even for the worst case (that is, when $\xi = 45^\circ$) about 91 percent of the accumulated momentum can be dissipated during darkness after the roll-only maneuver.

Slight changes in the reorientation intervals can equalize the daylight and darkness areas under the X_B -torque curve unless precluded by some mission constraint. Also, a slight offset from the sun-pointing position during daylight can accomplish the same result. This offset effect is shown by the dashed curves in figures 2 and 3 for the case $\xi_{\max} = 53.5^\circ$ in which the required offset is 3.5° . A maximum offset of about 12° is required when $\xi = 45^\circ$ and this maximum offset is still within the usual tolerance allowed for solar-panel pointing. For most situations, equalization of the areas need not be precise because the residual momentum can be kept within certain bounds by a series of iterative corrections during later orbits.

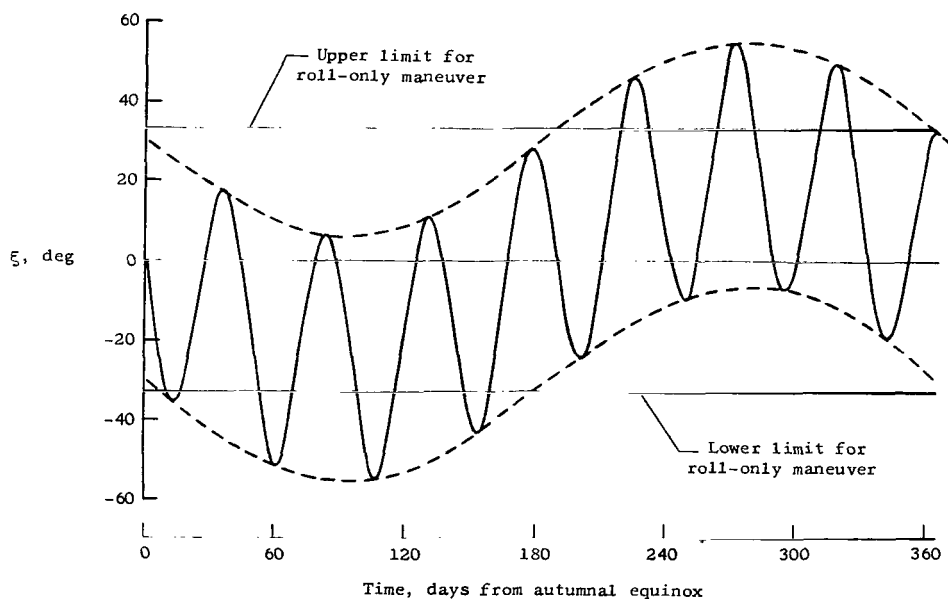


Figure 4.- Typical time history of ξ (angle the sun's rays make with orbit plane) during a 1-year period.

In some situations a pointing offset may be incompatible with the mission schedule, and thus it is of interest to know the times of the year when the roll-only scheme depends on the pointing-offset modification. The amplitudes of the torque components (see eq. (3)) are affected by the variation of ξ over the year; this variation is shown in figure 4 for the case where $\xi = 0$ at the autumnal equinox (September 23). The sinusoidal character of the curve is caused by the regression of the orbital plane at approximately 8 cycles per year. The dashed lines define the envelope of the curve for cases where $\xi \neq 0$ on September 23. The horizontal lines at $|\xi| = 33^\circ$ are the calculated limits for which all the momentum accumulated during the daylight portion of the orbit can be dissipated during the darkness portion by using a roll-only maneuver. This maneuver is then adequate for about 75 percent of the year since the curve in figure 4 is within the horizontal lines about this percentage of the time. Then, as already indicated, a pointing offset can be used to extend the roll-only maneuver to the remaining 25 percent of the year.

Because of the reduced power requirements and the simplicity of execution, the roll-only reorientation maneuver is a particularly attractive means of desaturating the momentum-exchange devices. However, the momentum-dissipation capability during part of the year may be too marginal to permit consideration of the roll-only maneuver exclusively. Provisions for an effective multiaxis maneuver would alleviate this situation and would also add much flexibility to the mission.

Multiaxis Reorientation

For cases where it is desirable to dissipate a large amount of momentum during a particular orbit, a number of multiaxis reorientation maneuvers can be performed near sundown which leaves the spacecraft in an orientation to take maximum advantage of the gravity-gradient moments during darkness. The aim of this maneuver is to place the long axis of the spacecraft approximately normal to the flight path and at a 45° angle with the orbit plane. In this orientation the maximum possible gravity-gradient torque can be used to dissipate the unidirectional component of the momentum accumulation.

A typical multiaxis reorientation maneuver is shown in figure 5. Rigidly mounted solar panels are shown on the spacecraft in order to indicate more clearly the direction of the sun; the sun's rays are normal to these panels in orbital positions from position ① to position ②. The postsundown maneuver (between positions ② and ③) involves rolling the spacecraft until the solar panels are normal to the orbit plane, yawing the spacecraft until the long axis of the spacecraft makes a 45° angle with the orbit plane, and establishing a rate on this long axis which keeps it approximately normal to the flight path. This orbital attitude is maintained for the next one-fourth orbit (between positions ③ and ④). During this interval the unidirectional or bias component of the gravity-gradient torque remains reversed (from its direction during daylight) at

approximately its maximum negative value. (See fig. 6.) Then between positions ④ and ① the second part of the total multiaxis reorientation maneuver takes place and restores the spacecraft to its initial attitude as it reaches orbit sunup. The presunup maneuver (between positions ④ and ①) illustrated in figure 5 is not necessarily an optimum maneuver but rather a typical maneuver which satisfies the additional constraints imposed by the solar panels.

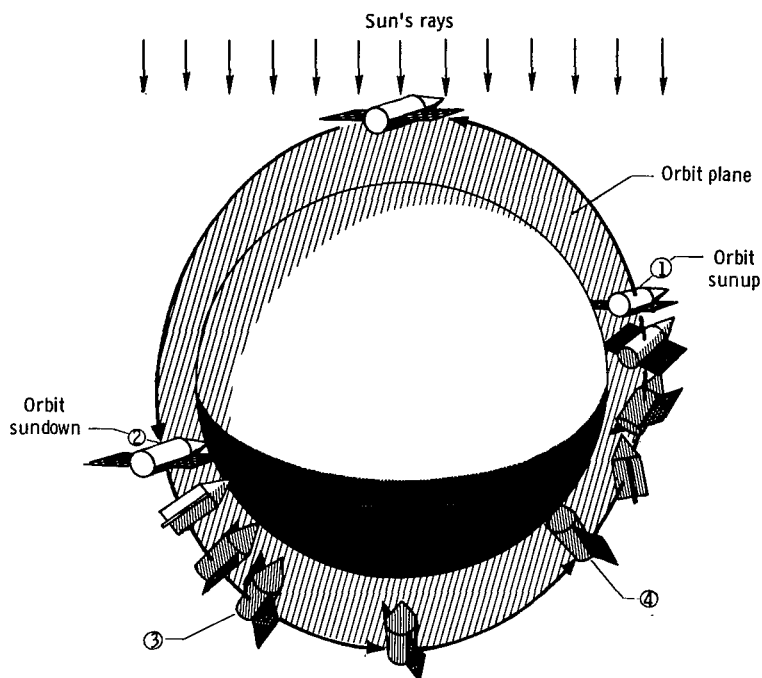


Figure 5.- Typical multiaxis reorientation maneuver.

The presunup maneuver is similar to the postsundown maneuver except it is performed in approximately reverse order. However, to arrive at sunup with the proper face of the solar panels to the sun, either a modified roll displacement or a spacecraft turnaround of 135° (as shown in fig. 5) can be incorporated into the maneuver. The turnaround requirement is, of course, a constraint due to the solar panels and would not be required in many other types of inertial tracking. However, it does probably involve the maximum body rates that would result from any selected multiaxis reorientation maneuver. For example, if the spacecraft were 12.19 meters (40 feet) long, the maximum acceleration in one end due to the turnaround would be on the order of $10^{-5}g$ and thus should not affect any zero-g experiments.

The results of the total multiaxis reorientation maneuver are shown in figure 6. The primary effect of the new orientation is to reverse the direction of the unidirectional

torque component and also to increase its magnitude greatly in the negative direction; this effect is shown by the Y_I -torque curve. The area under this curve (corresponding to momentum) indicates that the momentum-dissipation capability (negative area) during one orbital-darkness interval is sufficient to offset the momentum accumulation (positive area) during about 20 to 25 orbital daylight intervals. The curves in figure 6 are for the case of maximum unidirectional accumulation during daylight (that is, when the sun's rays are 45° incident to the orbital plane). The same large dissipation capability exists regardless of the accumulation conditions, and thus there are times when the accumulation from several hundred orbits may be dissipated during one darkness interval by using the multiaxis reorientation technique.

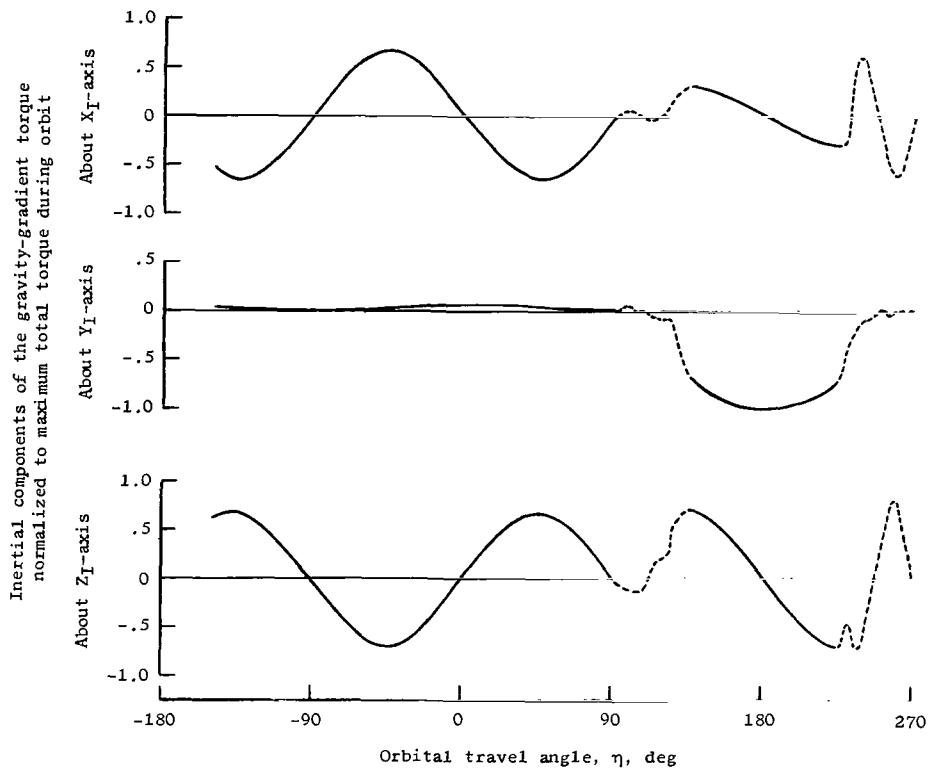


Figure 6.- Normalized gravity-gradient torque components for the case $\xi = 45^\circ$ showing effects of a typical multiaxis reorientation maneuver.

The X_I - and Z_I -torque components are also altered as indicated in figure 6, but both retain a generally cyclic nature. The short-dashed sections in the curves indicate where the multiaxis maneuvers take place; these dashed curves will tend to vary from maneuver to maneuver since there is no particular need for the maneuvers to be performed always in exactly the same manner as long as the final orientation is satisfactorily achieved.

Attitude Management

The cases of maximum torque disturbances and large-angle reorientation have been emphasized in the discussion thus far. However, during much of the year (fig. 4) the unidirectional momentum accumulation will be relatively small, and yet the large dissipation capability always exists (that is, one of the principal axes of the spacecraft can be tilted 45° to the local vertical whenever necessary). In certain situations when the sun's rays make a small angle with the orbit plane, the crew of the spacecraft may wish to use only part of the available momentum-dissipation capability. For example, when $\xi = 5^\circ$, the spacecraft can be rolled at sundown until the solar panels face about 10° out from the other side of the orbit plane during darkness. This 15° roll maneuver would require much less power than that for a larger angle reorientation. Under a different set of circumstances when the sun angle is 5° , the crew of the spacecraft may prefer to go several days without reorientation (because of an experiment) and then use a multiaxis maneuver to remove all the unidirectional momentum accumulation during one darkside interval. There may even be situations where the multiaxis maneuver could be used to introduce maximum "negative momentum accumulation" into the system prior to an extended attitude-hold interval; this scheme would effectively double the time available for maintaining a fixed inertial attitude without saturating the momentum-exchange system. Various other choices involving intermediate maneuvers and variation of the reorientation periods are also available.

No attempt was made in this study to optimize the exact starting points or the duration of any of the reorientation periods. However, some degree of year-round standardization seems to be feasible, especially for the roll-only maneuver. The following example is used to illustrate one approach to standardization for the case where solar panels must be pointed at the sun.

The ratio of daylight to darkness during each orbit varies over the year because of orbital-plane regression and the rotation of the earth about the sun. For a 200-nautical-mile orbit the darkness period varies from 1735 seconds to 2171 seconds or a difference of about 435 seconds. (See appendix C.) Reorientation periods of 218 seconds can then be selected which begin the year round at $\eta = 109.1^\circ$ and at $\eta = 236.7^\circ$. For the shortest day (orbital), these periods correspond to the 218 seconds just after sundown and before sunup, respectively. The full daylight period is thus available for pointing the solar panels at the sun. For the longest day, the reorientation periods correspond to the 218 seconds just before sundown and after sunup, respectively; for this condition the entire darkness period is then available for momentum dissipation. The length of this standard reorientation period is shown in terms of orbital travel angle in figure 2.

The control operations associated with any of the reorientation maneuvers should be well suited to manual control and iterative momentum management. In particular, maneuvering precision and timing are not very sensitive requirements for either the roll-only or multiaxis maneuvers and the procedures which must be followed to attain a particular attitude are not unique. Even the final orientation can be in error somewhat as long as the crew can observe some indication of the residual momentum in the system. For example, at appropriate times the crew can measure gimbal-angle displacements of the CMG controls and/or the speeds of the inertia wheels and then adjust subsequent maneuvers to reduce the momentum residue rather than add to it. The control commands can probably be initiated (or changed) by means of simple on-off electrical switching; thus, the attitude management tasks should become fairly trivial after the crew has established a familiarity with the operations.

CONCLUSIONS

In general, the technique of attitude control by means of planned momentum exchange between a system of inertia wheels or control-moment gyros and the space station seems to be feasible for long-term orbiting missions (for example, space stations). The primary feature of the technique proposed herein is the elimination of the requirement for using the attitude jets for desaturation of the control devices, and thus significantly reducing onboard fuel-storage requirements (or frequency of resupply by ferry). The following specific conclusions were derived from the results of this study:

1. The bias portion of the gravity-gradient disturbance can be associated with the spacecraft's axis of least inertia (usually roll) by alining this axis along the local vertical at orbit sunup.
2. All the bias angular momentum (due to the gravity-gradient torque) which will accumulate in the momentum-exchange system during one orbital-daylight interval can be dissipated during one orbital-darkness interval by properly reorienting the spacecraft in the gravity field. This reorientation can be accomplished with a roll-only maneuver during about 75 percent of the year. By using a small attitude-pointing offset from the reference direction (sun) during orbit daylight, the required dissipation can still be accomplished with the roll-only maneuver during the other 25 percent of the year. The capability of always dissipating much more angular momentum during darkness than will accumulate during daylight is available if a multiaxis reorientation maneuver is used.
3. With the multiaxis reorientation capability, the reorientation maneuver will be required not more than about once per earth day (24 hours); at certain times of the year it may not be needed for weeks at a time (depending on the sizing of the momentum-exchange devices and the position of the sun).

4. The reorientation maneuvers should not produce significant spacecraft angular-motion effects; the maximum body accelerations resulting from any of the maneuvers should not exceed $10^{-5}g$.

5. A great deal of flexibility and pilot volition can be incorporated into the proposed attitude control technique. Large tolerances for poorly executed reorientation maneuvers can be established since exact timing and a high degree of maneuvering precision are not mandatory. Errors which are incurred during one orbit can be removed on some later orbit by adjusting the subsequent maneuvers accordingly.

6. Some degree of year-round standardization can be worked out for the reorientation maneuvers, especially for the roll-only maneuver.

The technique discussed in this report is not limited to any initial orientation of the spacecraft, direction of attitude pointing (such as the sun), spacecraft configuration or type of momentum-exchange devices. At present, no inertia wheels or control-moment gyros large enough to maintain space-station attitude control have been built and tested. However, conceptual designs are being studied and wheels are being sized to particular space-station configurations. The proposed attitude-control techniques should only involve tailoring the existing hardware concepts to a modified set of operational requirements rather than adding any additional hardware to the spacecraft.

Langley Research Center,

National Aeronautics and Space Administration,

Langley Station, Hampton, Va., November 24, 1965.

APPENDIX A

GRAVITY-GRADIENT TORQUE EQUATIONS

A variety of transformations can be developed to express the long-term variations of the gravity-gradient torque in appropriate inertial and quasi-inertial reference systems. These equations would be, however, rather unwieldy and for the purposes of the present study, it is sufficient to consider the variations during a single orbit of the spacecraft.

Let X_I , Y_I , and Z_I be a right-handed reference set of Cartesian coordinates oriented in space such that the Z_I -axis is always sun-pointed. (See fig. 1.) The period of rotation of the Z_I -line about the sun is 365 days; thus, the direction of this line is essentially constant during any one orbit. Let \bar{R}_S be the position vector of the spacecraft and \hat{R}_S the associated unit vector; \bar{R}_S can be resolved into components by using the following direction cosines:

$$\left. \begin{aligned} l_{x'} &= \hat{R}_S \cdot \hat{X}_I = \sin \xi \cos \eta \\ l_{y'} &= \hat{R}_S \cdot \hat{Y}_I = \sin \eta \\ l_{z'} &= \hat{R}_S \cdot \hat{Z}_I = \cos \xi \cos \eta \end{aligned} \right\} \quad (A1)$$

where ξ is the orbital inclination to the sun's rays and η is the orbital travel angle. (See fig. 1.) To express \bar{R}_S in terms of the body-axis system, X_B , Y_B , and Z_B , the following transformation and the direction cosines (eqs. (A1)) can be used to determine a second set of direction cosines:

$$\begin{bmatrix} l_x \\ l_y \\ l_z \end{bmatrix} = \begin{bmatrix} a_{1j} \\ a_{2j} \\ a_{3j} \end{bmatrix} \begin{bmatrix} \hat{R}_S \cdot \hat{X}_I \\ \hat{R}_S \cdot \hat{Y}_I \\ \hat{R}_S \cdot \hat{Z}_I \end{bmatrix} \quad (A2)$$

where $[]$ denotes a matrix and a_{ij} are elements of the transformation matrix generated by the θ, ψ, ϕ order of rotation.

From reference 5 the generalized gravity-gradient torque equation can be written as

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$$\bar{T} = - \frac{3g_e R_e^2}{R_s^3} \left[\hat{i}_B (I_{zz} - I_{yy}) l_y l_z + \hat{j}_B (I_{xx} - I_{zz}) l_x l_z + \hat{k}_B (I_{yy} - I_{xx}) l_x l_y \right] \quad (A3)$$

where l_x , l_y , and l_z are the direction cosines of \bar{R}_s with respect to the X_B -, Y_B -, and Z_B -axis, respectively. By using the transformation equation (A2), the torque equation (A3) can thus be expressed as a function only of ξ , η , and the spacecraft attitude about the sun ray passing through the spacecraft center of mass. This result is given in detail by equation (A4).

$$\begin{aligned} \bar{T} = & - \frac{3g_e R_e^2}{2R_s^3} \left[\hat{i}_B (I_{zz} - I_{yy}) \left(\sin 2\eta \left[\sin \xi \left(\sin \theta \cos \psi \cos 2\varphi + \frac{1}{2} \cos \theta \sin 2\psi \sin 2\varphi \right) \right. \right. \right. \\ & + \cos \xi \left(\cos \theta \cos \psi \cos 2\varphi - \frac{1}{2} \sin \theta \sin 2\psi \sin 2\varphi \right) \left. \right] + \cos^2 \eta \left[\sin^2 \xi \left[(\sin^2 \theta \right. \right. \\ & - \cos^2 \theta \sin^2 \psi) \sin 2\varphi - \sin 2\theta \sin \psi \cos 2\varphi \left. \right] + \cos^2 \xi \left[(\cos^2 \theta - \sin^2 \theta \sin^2 \psi) \sin 2\varphi \right. \\ & + \sin 2\theta \sin \psi \cos 2\varphi \left. \right] + \sin 2\xi \left[\frac{1}{2} \sin 2\theta (1 + \sin^2 \psi) \sin 2\varphi - \cos 2\theta \sin \psi \cos 2\varphi \right] \left. \right\} \\ & + \sin^2 \eta (-\cos^2 \psi \sin 2\varphi) \left. \right] + \hat{j}_B (I_{xx} - I_{zz}) \left\{ \sin 2\eta \left[\sin \xi (\sin \theta \sin \psi \cos \varphi \right. \right. \\ & - \cos \theta \cos 2\psi \sin \varphi) + \cos \xi (\cos \theta \sin \psi \cos \varphi + \sin \theta \cos 2\psi \sin \varphi) \left. \right] \\ & + \cos^2 \eta \left[\sin^2 \xi (\sin 2\theta \cos \psi \cos \varphi + \cos^2 \theta \sin 2\psi \sin \varphi) - \cos^2 \xi (\sin 2\theta \cos \psi \cos \varphi \right. \\ & - \sin^2 \theta \sin 2\psi \sin \varphi) + \sin 2\xi (\cos 2\theta \cos \psi \cos \varphi - \sin 2\theta \sin 2\psi \sin \varphi) \left. \right] \\ & + \sin^2 \eta (-\sin 2\psi \sin \varphi) \left. \right\} + \hat{k}_B (I_{yy} - I_{xx}) \left\{ \sin 2\eta \left[\sin \xi (\sin \theta \sin \psi \sin \varphi \right. \right. \\ & + \cos \theta \cos 2\psi \cos \varphi) + \cos \xi (\cos \theta \sin \psi \sin \varphi - \sin \theta \cos 2\psi \cos \varphi) \left. \right] \\ & + \cos^2 \eta \left[\sin^2 \xi (\sin 2\theta \cos \psi \sin \varphi - \cos^2 \theta \sin 2\psi \cos \varphi) - \cos^2 \xi (\sin 2\theta \cos \psi \sin \varphi \right. \\ & + \sin^2 \theta \sin 2\psi \cos \varphi) + \sin 2\xi \left(\frac{1}{2} \sin 2\theta \sin 2\psi \cos \varphi + \cos 2\theta \cos \psi \sin \varphi \right) \left. \right] \\ & + \sin^2 \eta (\sin 2\psi \cos \varphi) \left. \right\} \end{aligned} \quad (A4)$$

Note that each component in this equation contains both cyclic and unidirectional functions of the orbital-travel angle η . However, by letting $\psi = 90^\circ$, equation (A4) can be collapsed into the simplified form:

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$$\begin{aligned} \overline{T} = & -\frac{3g_e R_e^2}{2R_s} \left\{ \hat{i}_B (I_{zz} - I_{yy}) \left[-\sin 2(\xi - \theta - \varphi) \right] \cos^2 \eta + \hat{j}_B (I_{xx} - I_{zz}) \left[\cos(\xi - \theta - \varphi) \right] \sin 2\eta \right. \\ & \left. + \hat{k}_B (I_{yy} - I_{xx}) \left[-\sin(\xi - \theta - \varphi) \right] \sin 2\eta \right\} \end{aligned} \quad (A5)$$

Physically, this restriction requires the spacecraft to be oriented so that the roll axis X_B is aligned with the local vertical \bar{R}_s at orbit sunup and orbit sundown. In this orientation the torque component about the roll axis will be unidirectional whereas the torque components about the other axes will be cyclic. If the Z_B -axis is required to be sun-pointed while $\psi = 90^\circ$, then $\theta = \varphi = 0$ in equation (A5).

Consider the unidirectional torque component when only $\theta = 0$; the expression $\sin 2(\xi - \varphi)$ then determines the direction of this component. During the sun-pointing period, φ is also zero, and the direction is determined by ξ alone. This direction can be reversed by choosing φ so that $\xi - \varphi$ is opposite in sign from ξ . Thus, the unidirectional torque component can be reversed by a simple roll maneuver of the spacecraft.

As mentioned earlier, the sun was selected as a convenient reference direction. However, this development of the gravity-gradient torque is also valid for any other reference direction provided ξ is defined as the angle between the orbital plane and the reference direction.

APPENDIX B

CONSIDERATIONS FOR SIZING THE MOMENTUM-EXCHANGE WHEELS

The spacecraft's stability and control system must include momentum-exchange devices of sufficient size to store the angular momentum expected to accumulate during a given period and still have adequate reserve capability to effect desired attitude rates (of the spacecraft) at the end of this period. This requirement can be illustrated by considering some typical examples.

Let a typical cylindrical-shaped spacecraft with the following inertia ratios be subjected to the conditions associated with figure 2; that is,

$$\left. \begin{array}{ll} I_{xx}' = 0.10 & \xi = 53.5^\circ \\ I_{yy}' = 0.95 & \varphi_{\max} = 98.5^\circ \text{ in } \Delta t \\ I_{zz}' = 1.00 & \Delta t = 218 \text{ sec} \end{array} \right\} \quad (B1)$$

If the X_B -component of equation (3) is integrated between the limits $t = \frac{109.1}{\omega}$ seconds, the momentum accumulating in the X_B -wheel(s) can be determined from

$$\left. \begin{array}{l} H_{xw}' = 4.948 \frac{R_e^2}{\omega R_s^3} \text{ N-m-s} \\ H_{xw}' = 3.680 \frac{R_e^2}{\omega R_s^3} \text{ lb-ft-sec} \end{array} \right\} \quad (B2)$$

where H_{xw}' is the normalized momentum accumulation in the X_B -wheel(s). If $I_{zz} = 10^6 I_{zz}'$, the unidirectional momentum accumulation during daylight for a 200-nautical-mile orbit is about

$$H_{xw} = 10^6 H_{xw}' = 176.25 \text{ N-m-s} \quad (B2a)$$

The maximum unidirectional accumulation H_{xw} is about 184 N-m-s and occurs when $|\xi| = 45^\circ$. However, when the devices themselves are used to set up spacecraft attitude rates for reorientation, the maximum reorientation-command requirements are a more important wheel-sizing consideration than the momentum-accumulation requirements. The following discussion illustrates this feature.

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Consider an inertia-wheel system in conjunction with equation (B1). The inertia wheels must be accelerated to develop torques to rotate the spacecraft; this acceleration must necessarily come at the time when the wheel on the roll axis X_B has reached its largest spinning rate because of the daylight momentum accumulation. If the X_B -wheel has an inertia of about 1.3558 kg-m^2 (1 slug-ft^2), the momentum accumulation (for $\xi = 53.5^\circ$) will cause this wheel to be spinning at about 1200 revolutions per minute at sundown. Then to roll the spacecraft through 98.5° in 218 seconds, the X_B -wheel must be accelerated until a spinning rate of about 11 400 revolutions per minute (or a change of about 10 200 revolutions per minute) is achieved. These values are determined by assuming a spin-up time of 10 seconds and a decay time of 10 seconds. A corresponding square pattern (that is, instantaneous rise and decay times) would require a spinning-rate step change of about 9750 revolutions per minute.

For the roll-only maneuver, the Y_B and Z_B inertia wheels will not be required to impart reorientation-command torques. These wheels will only be required to have the capacity to exchange and store the momentum resulting from the cyclic gravity-gradient torque components. (See eq. (3).) Wheels on the Y_B - and Z_B -axes having 1.3558 kg-m^2 (1 slug-ft^2) moments of inertia will be required to spin up in both directions to about 14 000 and 15 300 revolutions per minute, respectively, in the maximum torque cases. The maximum cyclic torque case occurs for the Y_B -wheel when $\xi = 0$ and for the Z_B -wheel when $|\xi| = 53.5^\circ$. Thus, a set of three inertia wheels each having a moment of inertia of 1.3558 kg-m^2 appears to be marginal in size for even the roll-only maneuver. In particular, the wheels on the Y_B - and Z_B -axes need to be larger to preclude such high maximum spinning rates. Also, no consideration has been given to other external disturbances on the spacecraft, such as aerodynamic torques which are significant at low altitudes.

If the inertia-wheel system is replaced by a CMG system consisting of single-axis twin gyros mounted on each principal axis, control torques to rotate the spacecraft can be developed by electrically applying a torque to the spin axes of the gyros. For the roll-only maneuver the system momentum can be expressed as

$$\bar{H}_S + 2\bar{H}_{wy} \sin \beta = 0 \quad (B3)$$

where \bar{H}_S is associated with the unidirectional component of the gravity-gradient torque and $2\bar{H}_{wy} \sin \beta$ is the resultant of the momenta due to the twin gyros on the Y_B -axis which have been displaced β degrees away from the null positions in order to "absorb" \bar{H}_S . For the case where $\xi = 53.5^\circ$,

APPENDIX B

$$\left. \begin{aligned} H_{wy} \sin \beta &= \frac{1}{2} H_S = 87.4 \text{ N-m-s} \\ H_{wy} \sin \beta &= \frac{1}{2} H_S = 65 \text{ ft-lb-sec} \end{aligned} \right\} \quad (B3a)$$

If the wheels are spinning at a constant speed of 10 000 revolutions per minute and each wheel has a moment of inertia of about 0.6779 kg-m^2 (0.5 slug-ft^2), the maximum daylight momentum accumulation can be compensated for by displacing the spin axes of the gyros by $\beta = 7.1^\circ$. Then, to perform the roll-only maneuver of $\varphi = 98.5^\circ$ in 218 seconds, the spin axes must be displaced to about $\beta = 65^\circ$ and held during the period of momentum dissipation. Cross-coupling effects due to the high spin rate (10 000 revolutions per minute) of the wheels will tend to change these values of β somewhat, depending on the rate that the spin axis is being displaced. Note that very little reserve capability is left for compensation of disturbances other than the gravity-gradient torque.

The momentum-storage capacities of the twin gyros associated with the Y_B - and Z_B -axes must be at least an order of magnitude greater than those for the X_B -axis. Each wheel of a gyro pair will compensate for about half of this total when working in combination with its twin. If the maximum momentum storage requirement is about 2169 N-m-s (1600 lb-ft-sec),

$$H_{wz} \sin \beta = \pm 1084.5 \text{ N-m-s} \quad (B4)$$

A wheel with an inertia of 1.3558 kg-m^2 (1 slug-ft^2) spinning at 10 000 revolutions per minute will generate about 1420 N-m-s of impulse. Thus, the travel range of β for the twin wheels must be at least

$$\beta = \sin^{-1} \left(\frac{1084.5}{1420} \right) = \pm 49^\circ \quad (B5)$$

Then, based on these calculations, it appears that for a twin-gyro CMG system to be adequate for attitude control by using the roll-only maneuver, each of the six wheels must have a moment of inertia of at least 1.3558 kg-m^2 (for operation at 10 000 revolutions per minute).

When multiaxis maneuvering capability is superimposed upon the storage capacities of the wheels associated with the pitch (Y_B) and yaw (Z_B) axes, the wheel storage capacities must be increased to 6 or 7 kg-m^2 . Not only must the wheels be larger in size, but the power requirements are also increased. A "power penalty" (namely, an increase in mass due to an increase in size of the power supply) of several hundred kilograms per kilowatt is required for state-of-the-art equipment. Part of this increased weight is due to larger solar-cell panels which, in turn, will tend to increase aerodynamic disturbances

APPENDIX B

on the spacecraft. Thus, it appears that CMG attitude-control systems with their much smaller power requirements may be preferred over inertia-wheel systems in many cases. Also, the use of double-gimbaleed gyros (in place of twin gyros) may reduce the system weight.

GEOMETRIC DETERMINATION OF ORBITAL DAYLIGHT-DARKNESS RATIO

VIEW A

Orbit

Earth

P

P'

c

d

Sun's rays

VIEW B

Earth

Orbit

VIEW C

Orbit

Earth

P

P'

c

R_s

d

Darkness

daylight-darkness divisions or actual sunup and actual sundown if there were no diffraction. The vertical plane PP' containing one of these rays is established and its intersection with the earth is shown as a circle in view B (where the orbit appears as an edge view). The orbit is shown as a true circle in view C where plane PP' again appears as an edge view. Then by geometry,

$$d^2 = R_e^2 - c^2 \quad (C1a)$$

APPENDIX C

from view C:

$$c = R_S \sin \alpha \quad (C1b)$$

$$f = R_S \cos \alpha \quad (C1c)$$

from view B:

$$\sin \xi = \frac{d}{f} \quad (C1d)$$

Then to determine α ,

$$\sin^2 \xi = \frac{R_e^2 - R_S^2 \sin^2 \alpha}{R_S^2 \cos^2 \alpha}$$

$$R_S^2 \cos^2 \alpha \sin^2 \xi + R_S^2 (1 - \cos^2 \alpha) = R_e^2$$

$$R_S^2 \cos^2 \alpha (1 - \sin^2 \xi) = R_S^2 - R_e^2$$

$$\cos^2 \alpha = \frac{R_S^2 - R_e^2}{R_S^2 \cos^2 \xi}$$

$$\alpha = \cos^{-1} \frac{\sqrt{R_S^2 - R_e^2}}{R_S \cos \xi} \quad (C2)$$

From view C the angle α is shown to subtend half of the dark portion of the orbit and from view B the angle ξ is the angle the sun's rays make with the orbit plane. Using $|\xi| = 53.5^\circ$ in equation (C2), the shortest night for the 200-nautical-mile orbit corresponds to $\alpha = 56.66^\circ$ and thus the dark time is

$$(t_d)_{\min} = \frac{2\alpha}{360}(5511) = 1735 \text{ sec} \quad (C3a)$$

where 5511 seconds is the orbital period. Similarly, the longest night occurs when $\xi = 0$; that is,

$$(t_d)_{\max} = \frac{2(70.92)}{360}(5511) = 2171 \text{ sec} \quad (C3b)$$

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